1 Show that, under a Lorentz boost: $E\,\delta^3(\vec{p}-\vec{k}) = E'\,\delta^3(\vec{p}\,'-\vec{k}\,')$

2 Starting from the Heisenberg Picture form of the scalar field operator (eq. 53.3 of the lecture notes), obtain the form of the Schrödinger Picture form:

$$\hat{\phi}_S(\vec{x}) = e^{-i\hat{H}t}\,\hat{\phi}_H(\vec{x},t)\,e^{i\hat{H}t}$$

3 Let's continue exercise 2 from the previous lecture. We are still dealing with the complex scalar field:

(Peskin 2.2)

$$S = \int d^4x \left(\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \right)$$

Using the results already obtained for it:

3 a Show that this theory has two different sets of excitations, but both satisfy the relativistic dispersion relation for the same mass "m"

(in other words, we have two "types" particles with the same mass)

3 b The Lagrangian is invariant by changes on the complex phase of the field, show that Noether's Theorem thus leads to the conservation of the following quantity:

$$Q = \int d^3x\, \frac{i}{2}\left(\phi^* \pi^* - \pi \phi \right)$$

(let's name this one "charge", but to show any relation with electric charge we must first have electromagnetic fields, which is still in the future)

3 c Write the "charge" operator above in terms of creation and annihilation operators and show that the two types of particles have oposite charges.